

SHORT NOTES

MASS BALANCE AND THERMAL REGIME OF A CRATER GLACIER AT USHKOVSKII  
VOLCANO

Ya.D. Muravyev and A.N. Salamatin

Institute of Volcanology, Far East Division, USSR Academy of  
Sciences, Petropavlovsk-Kamchatskii;  
Kazan University, Kazan

A thermal model has been constructed for a steady-state glacier in the active crater of Ushkovskii Volcano. Analysis of ice mass balance components has revealed elevated heat flow (mean value  $10 \text{ W/m}^2$ ) in the summit crater which has remained nearly constant over the last 40 years. The measured accumulation rate and temperature distribution in the snow and firn body in the middle of the Gorshkov crater suggest the existence of a considerable uplift (a small embedded crater) overlain by the glaciers. The formulas proposed in this paper can be used to evaluate critical state parameters for unsteady ice masses on the slopes of Klyuchevskoi Volcano.

(Received December 25, 1987)

Ushkovskii Volcano, one of the large Kamchatka volcanoes, is situated in the northwestern corner of the Klyuchevskoi volcanic

group. Its huge complex volcanic cone of Pleistocene age measures some  $370 \text{ km}^3$  in volume (together with Krenovskii Volcano). The summit area of  $43 \text{ km}^2$  is covered by ice and the ice filling a  $24.2 \text{ km}^2$  caldera on top of the volcano serves as a source area for many large Kamchatka glaciers: Bil'chenok, Erman, Bogdanovich and others. The volume of ice in the caldera has been estimated at about  $5 \text{ km}^3$ . In the case of a subglacial eruption, the ice will melt and large quantities of water will form enough to sweep over the countryside in the form of lahars and jökulhlaups. The last eruption of the volcano is believed to have occurred in the spring of 1880 [7]. The present-day knowledge of the volcano is the rising temperature of the rims of the summit craters and the existence of geothermal caves within the glacier.

Research in 1983-1986 showed that the snow and ice on the flat summit at 3900 m height above sea level can only melt due to volcanic heat, ablation being a mere 5-10% of the annual accumulation. The mean accumulation rate is about  $1000 \text{ kg/m}^2$ , ranging between  $400 \text{ kg/m}^2$  on the outer slope of the crater and  $1300 \text{ kg/m}^2$  in its centre. The temperature of the snow and firn mass is  $-18^\circ\text{C}$  at 10 m depth. This value can be recognised as the mean annual air temperature  $\theta_0$  at that altitude. This temperature is consistent with the mean temperature in the town of Klyuchi ( $+1.5^\circ\text{C}$ ) which was reduced to the summit crater altitude using a temperature gradient of  $5.3^\circ\text{C/km}$  for the Klyuchevskoi area [6]. The amount of cold which is stored in the glacier is thus sufficient for the melt water to freeze again at depths of 20-30 cm below the surface.

Judging by the climatic environment and experimental measurements, this portion of the source area for Bil'chenok glacier can be classified as a recrystallisation and regelation zone of ice production [3]. Similar environments are known for the Pamir firn plateau (at altitudes above 5800 m) and for the Mirnyi Observatory, 40-50 km from the Antarctic coast [4].

The data of aerial surveys conducted since 1948 and the field work do not reveal appreciable changes on the glacier surface, indicating that the accumulation rate at the top and the bottom ablation are equal, that is, that the ice mass balance is nearly zero. Measurements in a geothermal cave have yielded the rates of ice mass transport: the horizontal velocity of ice motion from the crater is about 0.2 m/yr at the outer rim and slightly greater, 0.33 m/yr, at the inner. The vertical subsidence velocity for the ice layers above the crater is 0.33 m/yr. It follows that the ice flow from the 800 m crater is a mere 1-1.5% of the ablation loss. The mean annual ice accumulation for the past 40 years is 0.5 million cubic metres of the water equivalent. These data can be used in solving some glaciological problems in areas of active volcanism.

#### THERMAL BEHAVIOUR OF A STEADY-STATE GLACIER

With a view to a further interpretation of the field observations and experiments, we shall examine the thermal behaviour of a glacier for the steady state of the ice mass, which is the case of the glacier in the large crater of Ushkovskii Volcano. The process is characterised by a vertical ice transfer caused by snow accumulation at the top and melting at the bottom, the latter compensating a possible increase in height ( $h$ ) due to precipitation. An analysis of snow density distribution in boreholes shows that the most significant change in density occurs within the first metre where  $\rho$  increases rapidly from 250 to 400 kg/m<sup>3</sup> and then grows slowly to reach 600 kg/m<sup>3</sup> at 13 m depth (Figure 1). The mean density value in the interval of interest, 1 to 8 m, can be assumed to be 450 kg/m<sup>3</sup> and this value can be used to determine the other characteristics of the snow cover. The presence of thin seasonal layers of infiltration ice helps evaluate the accumulation rate (velocity of motion) for firn within a 10 metre interval below the surface,  $v=2.7$  m/yr ( $0.86 \times 10^{-7}$  m/s). This value was

nearly constant over the last three years and consistent with the invariable annual precipitation at the Klyuchi weather station for the same period of time.

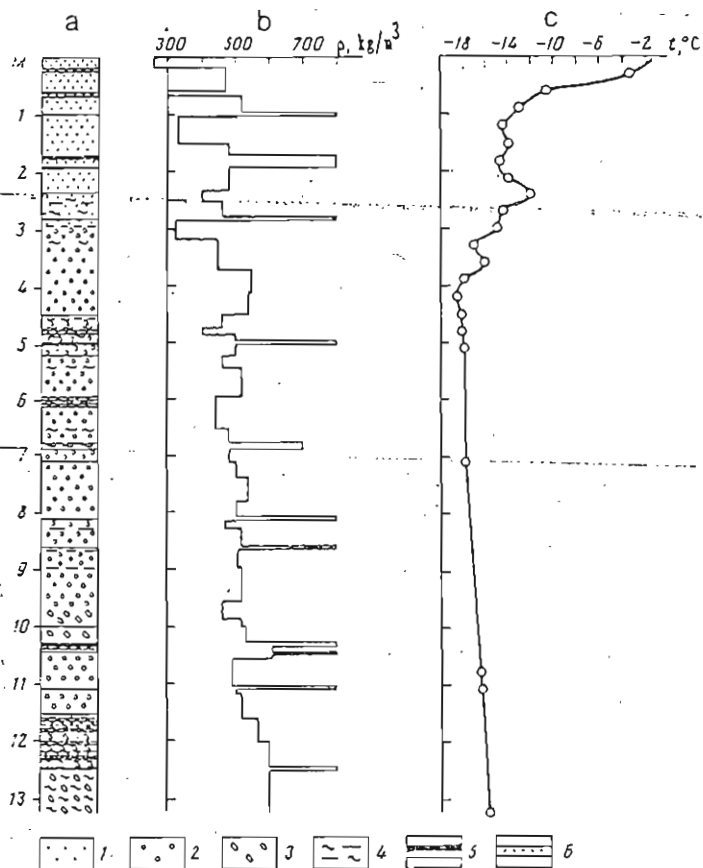


Fig. 1. Stratification (a), density (b), and temperature distribution (c) of a snow and firn mass from the Gorshkov crater: 1 - fine-grained snow; 2 - medium-grained firn; 3 - coarse-grained firn; 4 - ice lenses and intermittent layers; 5 - infiltration ice; 6 - ash beds.

When we determined the mean annual temperature distribution in the snow and firn mass we took into consideration the fact that seasonal fluctuations do not affect the lower 10-13 m depth interval reached by a BH-1 borehole. Besides, we assumed, according to data from [5], the thermal conductivity  $\lambda$  of the upper 1-m snow layer to be lower by a factor of 5. The mean annual temperature distribution is shown in Figure 3 (dashed line). The relevant mean thermal gradient  $G$  is 0.175 K/m. The theoretical temperature distribution for a deep borehole was found from

$$\theta = \theta_0 + GZ + A_0 e^{-z \left( \sqrt{\frac{\omega}{2\kappa}} - \frac{v}{2\kappa} \right)} \cos(\omega\tau - z \sqrt{\omega/2\kappa}). \quad (1)$$

Here  $Z$  is the depth below the surface,  $\kappa$  the thermal diffusivity for snow,  $A_0$  the range of seasonal temperature variation,  $\omega$  the frequency of temperature changes, and  $\tau$  is the time. This relation is a simplified solution of the temperature distribution problem that has been examined in [1] without constraints for the initial conditions, its uncertainty being the order of  $\theta(v^2/8\kappa\omega)$ .

The best fit between (1) and the temperature measurements was obtained for  $\sqrt{\omega/(2\kappa)} \approx 0.43 \text{ m}^{-1}$ , corresponding to  $\omega = 2 \times 10^{-7} \text{ s}^{-1}$  and  $\kappa = 5.4 \times 10^{-7} \text{ m}^2/\text{s}$ . Taking the specific heat capacity for ice  $C = 2000 \text{ J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$  and  $\rho = 450 \text{ kg}/\text{m}^3$  we found thermal conductivity  $\lambda \approx 0.49 \text{ W}/\text{m} \cdot \text{K}^{-1}$ . Our results are in good agreement with data from [5] and consistent with the measurements of  $\lambda$  and  $\kappa$  made by D.N. Dmitriev and R.N. Vostretsov for cores from a borehole at the Vostok station in the Antarctic for similar values of snow and firn density (Figure 2).

One can notice systematic deviations of the observed temperatures from the theoretical curve in Figure 3 which cannot be accounted for by measurement errors. They are obviously periodic and decrease with depth. The frequency of their spatial variations  $\sqrt{\omega/(2\kappa)}$  is nearly four times the frequency of the vertical changes due to the seasonal temperature variation at the surface.

It follows that they are most likely associated with short-term temperature changes (with a period of 16 times as short) which occur within ten-day intervals or even weeks and measure 3-5°C.

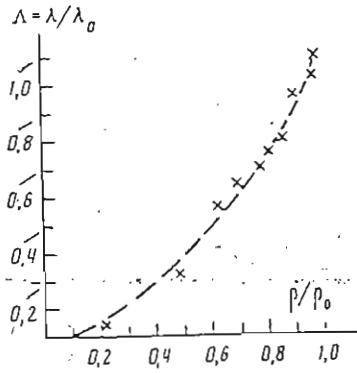


Fig. 2. Relative thermal conductivity of snow-and-firn deposits as a function of relative density. Crosses indicate borehole measurements.

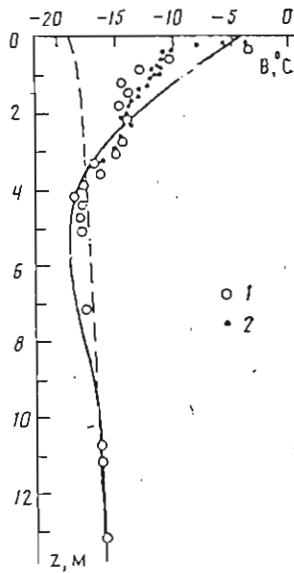


Fig. 3. Temperature measurements in boreholes and pits on the volcano summit. 1 - borehole BH-I; 2 - pit III.

We are now going to describe the temperature field within a glacier. We put the coordinate origin at the glacier bed, the x-axis pointing upward. Let  $J_h$  be the accumulation rate for ice-and-firn deposits,  $q_0$  the heat flow at the bed,  $J_b$ ,  $\theta_b$ , and  $L_b$  the accumulation rate, temperature and latent heat capacity of ice in the basal layer, respectively,  $\lambda_0$  the typical value of thermal conductivity  $\lambda$ , for instance, that for pure ice, and  $\tau$  the time.

In that case the problem of determining the glacier temperature field  $\theta$  and thickness  $h$  when expressed in dimensionless form can be stated as follows:

$$\begin{aligned} \frac{d}{dx} \left( \Lambda \frac{dv}{dx} \right) + K_\theta \frac{dv}{dx} &= 0, \quad 0 < x < l; \\ \text{a) } x = 0, \quad \frac{dv}{dx} &= K_J - 1, \quad \tilde{v} = 1; \\ \text{б) } x = l, \quad v &= 0. \end{aligned} \quad (2)$$

Here

$$\begin{aligned} v &= \frac{\theta - \theta_0}{\theta_b - \theta_0}, \quad x = \frac{q_0 x}{\lambda_0 (\theta_b - \theta_0)}, \quad l = \frac{q_0 h}{\lambda_0 (\theta_b - \theta_0)}, \\ \Lambda &= \lambda / \lambda_0, \quad K_\theta = \frac{c J_h (\theta_b - \theta_0)}{q_0}, \quad K_J = \frac{L_b J_h}{q_0}. \end{aligned} \quad (3)$$

The criterion  $K_\theta$  characterises the effect of mass transfer on the thermal behaviour of the glacier and  $K_J$  is the ratio of a mass accumulation rate to the maximum possible rate of ice melting at the bottom owing to heat rising from beneath the volcano. These criteria are the main parameters which describe the growth of a glacier and its steady state under constant climatic and geothermal conditions ( $\theta_0$ ,  $J_h$ ,  $q_0 = \text{constant}$ ). It can be demonstrated that with  $K_\theta + K_J \geq 1$  the growth of the glacier thickness with time is unlimited and that dynamic equilibrium for a finite thickness  $l$  (without discharge over the crater rim) cannot be achieved by the compensation of ice accumulation by basal ice melting along. A steady-state solution is only possible for  $K_\theta + K_J < 1$ . In that case

the solution of (2) is given by

$$\int_0^l \frac{dx}{\Lambda(x)} = -\frac{1}{K_0} \ln \left( 1 - \frac{K_0}{1 - K_J} \right), \quad (4)$$

$$v(x) = 1 - \frac{K_J - 1}{K_0} \left[ \exp \left( -K_0 \int_0^x \frac{dx}{\Lambda} \right) - 1 \right].$$

We will use these results to evaluate the ice conditions in a volcanic area.

#### THERMAL CONDITIONS OF A GLACIER IN THE MIDDLE OF GORSHKOV CRATER, USHKOVSKII VOLCANO

Let us analyse the thermodynamic state of an ice mass in the Ushkovskii crater. Our computation will be based on the experimental data obtained in the 1986 expedition on Klyuchevskoi and on the results of the above data processing.

Using the subscript h to mark the parameters that refer to the top layer of the ice-and-firn deposits, we have

$$\begin{array}{ll} \rho_0 = 920 \text{ kg/m}^3 & \lambda_0 = 2.2 \text{ W/(m.K)} \\ C = 2 \text{ kJ/(kg.K)} & L_0 = 335 \text{ kJ/kg} \\ \theta_b - \theta_0 = 19 \text{ K} & \rho_h = 450 \text{ kg/m}^3 \\ \lambda_h = 0.49 \text{ W/(m.K)} & G_h = -(\partial\theta/\partial x)|_{x=h} = 0.175 \text{ K/m} \end{array}$$

The rate of snow accumulation based on observations in pits and boreholes is  $v_h = 0.86 \times 10^{-7}$  m/s which corresponds to  $J_h = 3.9 \times 10^{-5}$  kg.m<sup>-2</sup>.s<sup>-1</sup>. With these values in hand and assuming the glacier in the Gorshkov crater to be in a state close to dynamic equilibrium (i.e., in the steady state), we will use (4) to find its thickness l (or h) and the heat flow  $q_0$ , and hence the temperature distribution v (or  $\theta$ ). The differentiation of (4) with respect to x,  $x=l$ , gives

$$-\frac{\partial v}{\partial x} \Big|_{x=l} = \frac{K_J + K_0 - 1}{\Lambda(l)}$$



Passing to dimensional quantities in this equality by using (3), we have

$$q_0 = J_h [C (\theta_b - \theta_0) + L_b] + \lambda_b \Gamma_b,$$

$$G_b = -(\partial \theta / \partial x) |_{x=b}.$$

Corresponding to the original data given above is the value  $q_0 \approx 14.63 \text{ W/m}^2$ . Obviously, this estimate is large determined by the value of  $J_b$  and depends on the reliability of our knowledge of the rate of ice accumulation at the glacier. The mean value for the Gorshkov crater is

$$J_b \sim 1.85 \times 10^{-5} \text{ kg/(m}^2 \cdot \text{s)} \quad (v_b \approx 1.3 \text{ m/yr})$$

which gives  $q_0 = 7 \text{ W/m}^2$ . The respective criteria for the two cases are  $K_b = 0.101$  and  $0.1006$ ;  $K_j = 0.893$  and  $0.887$ .

In both cases the sum  $(K_j + K_b)$  is little different from 1, which implies that the heat flow is almost wholly used to heat and melt the ice, except the amount of  $1 - (K_j + K_b)$  which passes through the ice and goes to the atmosphere (less than 1%).

When expressed in dimensional quantities to compute a steady-state glacier thickness, equation (4) takes the form

$$\int_0^h \Lambda^{-1} dx = \frac{\lambda_0}{C J_h} \ln \left[ 1 + \frac{C J_h (\theta_b - \theta_0)}{\lambda_h G_h} \right]. \quad (5)$$

Note that the right-hand side of (5) is essentially dependent on  $G_b$  and the exact determination of this parameter is important to compute  $h$ .

In addition, to estimate  $h$  we need to know the relationship  $\Lambda(x) \sim \lambda(x)/\lambda_0$  or its reciprocal. As a rule, for glaciers dominated by recrystallisation as a mechanism of ice formation,

$$\Lambda^{-1} = 1 + a e^{-b(h-x)}. \quad (6)$$

Substituting this in equation (5) and integrating, we find

$$h - \frac{a}{b} e^{-bh} = \frac{\lambda_0}{C J_h} \ln \left[ 1 + \frac{C J_h (\theta_b - \theta_0)}{\lambda_h G_h} \right] - \frac{a}{b}. \quad (7)$$

Realistic values of  $h$  make the second term in the left-hand side negligibly small in most cases.

To plot the relationship  $\Lambda^{-1}$  for the ice-and-firn deposits at Ushkovskii Volcano we used the values of thermal conductivity and density for snow as given in [5], the experimental data of D.N. Dmitriev and R.N. Vostretsov who measured thermal conductivity on ice cores from the Antarctic, ice core density determinations by Lipenkov [8], and our results considered in the preceding section. Note that the data scatter was fairly small. The resulting plot of  $\lambda/\lambda_0 = \Lambda^{-1}$  as a function of the relative density of ice-and-firn deposits  $\rho/\rho_0$  is presented in Figure 2.

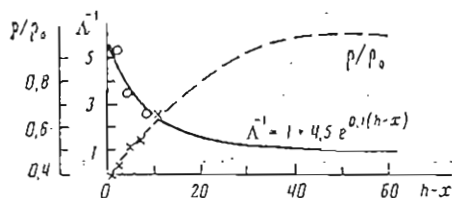


Fig. 4. Theoretical variations of relative density and relative thermal conductivity with depth for the snow-and-firn deposits in the Gorshkov crater. Crosses show ice density measurements in pits.

The hypothetical curve of  $\lambda/\lambda_0$  as a function of depth ( $h-x$ ) is shown as a dashed line in Figure 4, where crosses give density determinations in boreholes at Ushkovskii Volcano. The open circles show  $\Lambda^{-1}(x)$  based on the distribution of  $\lambda(x)/\lambda_0$  and on the relation in Figure 2. It is quite satisfactorily fitted (the solid line in Figure 4) by a function of the form (6) with  $a=4.5 \text{ m}^{-1}$  and  $b=0.1 \text{ m}^{-1}$ . We thus have  $a/b=45 \text{ m}$  for the glacier in the Gorshkov crater. In that case with  $J_h=3.9 \times 10^{-5} \text{ kg}/(\text{m}^2 \cdot \text{s})$  equation (7) yields  $h=38 \text{ m}$ . The glacier thickness is obviously underestimated by this value. A geometrical calculation gives a depth of about 200 m to the crater flow in the middle of the crater using

the crater diameter 800 m and the slope of about  $30^\circ$ . This discrepancy indicates that the rates of ice accumulation measured in the boreholes are misleading and seem to be considerably above the mean value for the area of study. However, even with  $J_h = 1.85 \times 10^{-5}$  kg/(m<sup>2</sup>.s), equation (7) gives  $h = 87$  m. This value is close to a possible mean crater depth and to the corresponding mean glacier thickness. It should nonetheless be stressed that  $h = 87$  m is less than a half of the crater depth at its centre as given by the slope of its inner sides. For comparison, the North Crater in the summit caldera of Wrangell Volcano, Alaska, which is similar in size to the Gorshkov crater has a maximum depth of 160-180 m at the centre [2]. Two possible explanations are the existence of a central uplift (small embedded crater?) or a debris accumulation in the crater (less likely). The former hypothesis is the more likely because the Gerts crater situated nearby is a small embedded crater lying in a large crater which is similar to the Gorshkov crater in size and is overlain by a glacier. A likely explanation is a complex combination of these factors with a large range of the accumulation rate in the Gorshkov crater from  $J_h \approx 10^{-5}$  kg/(m<sup>2</sup>.s) in the southwest to  $J_h \approx 5 \times 10^{-5}$  kg/(m<sup>2</sup>.s) in the northeast and a partial redistribution of ice within the crater in the opposite direction with the mean background value of heat flow being approximately  $10 \text{ W/m}^2$ .

These estimates can be improved as more data are collected on the near-surface temperature gradient  $G_b$  and thermal conductivity  $\lambda_b$  in the snow and firn mass and on the rate of ice accumulation  $J_b$  from many sites disturbed over the crater area.

The substitution of (6) in (4) with criteria (3) in mind yields a simple formula for temperature distribution in a glacier

$$\theta(x) = -\frac{q_0 - LJ_h}{CJ_h} \left\{ 1 - e^{-\frac{CJ_h}{\lambda_b} x} \left[ x + \frac{a}{b} (e^{-b(\lambda-x)} - e^{-b^2}) \right] \right\}. \quad (8)$$

where  $\theta_0 = 0^\circ\text{C}$ . The factor  $(q_0 - LJ_1)$  can be replaced by  $CJ_1 = (\theta_0 - \theta_0) + \lambda_1 C_1$  which is equal to it.

The temperature distribution in a glacier body computed from (8) for the accumulation rates considered above is plotted in Figure 5. The S-shaped curves of temperature distribution are due to a combination of lower thermal conductivity in the upper layers with convective heat transfer.

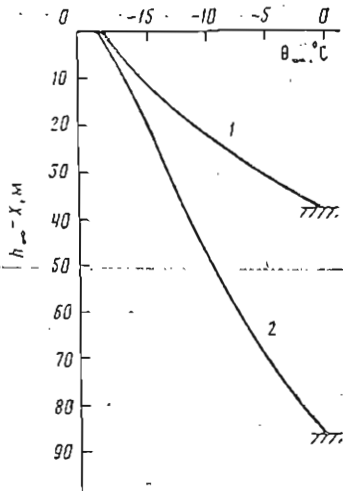


Fig. 5. Temperature distribution in the body of the crater glacier for two values of heat flow and accumulation rate.

### CONCLUSION

The low contribution of solar heat to the total ablation, and hence to the annual ice mass balance, makes it possible to use the crater glacier of Ushkovskii Volcano as a simple model to compute thermal characteristics of inaccessible, dynamically unstable glaciers at Klyuchevskoi Volcano. The study of the thermal conditions of the snow-and-firn mass has shown that the glacier has been in a quasisteady state over the last decades and that a specific temperature distribution exists in its body that does not

vary with time. It can therefore be concluded that the heat flow at Ushkovskii Volcano exhibited little variation during a long period of time.

Deep borehole thermometry has been examined to estimate the contributions of the main heat balance components using Budd's [1] simplified technique with due allowance for seasonal variations. This approach gave  $\lambda_h$  and  $J_h$  and corroborated the previous results through the comparison of theoretical and experimental temperatures. Two versions of temperature distribution within a glacier body have been considered. Both of them demonstrated that all formulas are strongly dependent on the accumulation rate. The accumulation rates measured in boreholes gave a glacier thickness as small as 40 m. Assuming that  $J_h$  may vary from one crater locality to another its value was reduced by half and the computation gave a thickness of 90 m. As indicated by snow survey data, ice accumulation is not uniform over the crater area and an intra-crater mass flow must exist from northeast to southwest at a rate of at least 2 m/yr.

The total heat flow of the Gorshkov crater computed by the calorimeter method is 9 MW, at least 80% of this amount being consumed to melt the ice and heat the ice water.

#### REFERENCES

1. W.P. Budd, The Dynamics of Ice Masses, Antarctic Division, Department of Supply, Melbourne, 1969.
2. C. Benson et al.; *Giyatsiologicheskii Iissledovaniya*, 27, 114-133 (1985).
3. V.M. Kotlyakov, Ed., Glossary of Glaciology, GIMIZ, Leningrad, 1984.
4. M.B. Dyurgerov and N.A. Urumbaev, Glaciological Studies of the Pamir Firn-and-Ice Plateau, *MGI*, 31, 30-38 (1977).
5. V.P. Isachenko, V.A. Osipova and A.S. Sukomel, Heat Transfer, *Energiya*, Moscow, 1969.

6. Ya.D. Murav'ev, in: The Geography of Kamchatka, Petropavlovsk-Kamchatskii, 9, 30-40 (1985).
7. B.I. Piip, Klyuchevskoi Volcano and its Eruptions in 1944-1945 and in the Past, Izd-vo Akad. Nauk SSSR, Moscow, 1956.
8. A.N. Salamatin et al., in Antarctic: Commission Reports, Nauka, Moscow, 24, 99-105 (1985).